CSC721 **Algorithms** **Spring 2013**

Asymptotic Measures

Let f and g be functions with $f : \mathcal{N} \to \mathcal{R}^+$ and similarly, $g : \mathcal{N} \to \mathcal{R}^+$

Definition: (Big O)

We say $f(n) \in \mathcal{O}(g(n))$ if and only if there exists $c > 0$ and $N_0 > 0$ such that

$$|f(n)| \leq |cg(n)| \quad \text{for all } n \geq N_0$$

Definition: (Little o)

We say $f(n) \in o(g(n))$ if and only if for all $c > 0$ there exists $N_0 > 0$ such that

$$|f(n)| \leq |cg(n)| \quad \text{for all } n \geq N_0$$

if $g(n)$ is non-zero, then an equivalent (and often more useful) definition is given by:

$$\lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| = 0$$

Definition: (Big Theta)

We say $f(n) \in \Theta(g(n))$ if and only if $f(n) \in \mathcal{O}(g(n))$ and $g(n) \in \mathcal{O}(f(n))$.

Definition: (Big Omega)

We say $f(n) \in \Omega(g(n))$ if and only if there exists $c > 0$ and $N_0 > 0$ such that

$$|f(n)| \geq |cg(n)| \quad \text{for all } n \geq N_0$$

Some properties of Big O

1. For any constant c, we have: $cf(n) \in \mathcal{O}(f(n))$.
2. If $f_1(n) \in \mathcal{O}(g(n))$ and $f_2(n) \in \mathcal{O}(g(n))$ then $(\, f_1(n) + f_2(n)\,) \in \mathcal{O}(g(n))$.
3. If $f_1(n) \in \mathcal{O}(g_1(n))$ and $f_2(n) \in \mathcal{O}(g_2(n))$ then $f_1(n)f_2(n) \in \mathcal{O}(g_1(n)g_2(n))$.
4. If $f(n) \in \mathcal{O}(g(n))$ and $g(n) \in \mathcal{O}(h(n))$ then $f(n) \in \mathcal{O}(h(n))$.
5. $f_1(n) + f_2(n) \in \mathcal{O}(\max(\, f_1(n), f_2(n)\,))$