

LUP Decomposition: Pictorial Interpretation

$$\begin{matrix} n \\ \boxed{A} \\ m \end{matrix} = \begin{matrix} m \\ \boxed{L} \\ m \end{matrix} \cdot \begin{matrix} n \\ \boxed{U} \\ m \end{matrix} \cdot \begin{matrix} n \\ \boxed{P} \\ n \end{matrix}$$

LUP - Factorization

$$\begin{matrix} p \\ \boxed{A} \\ m \end{matrix} = \begin{matrix} p \\ \boxed{B} \\ m/2 \\ \boxed{C} \\ m/2 \end{matrix}$$

Step 1: Partition Matrix A into Blocks B and C

$$\begin{matrix} p \\ \boxed{A} \\ m \end{matrix} = \begin{matrix} m \\ \boxed{L_1} \\ m/2 \\ \boxed{I} \\ m/2 \end{matrix} \cdot \begin{matrix} p \\ \boxed{U_1} \\ m/2 \\ \boxed{D} \\ m/2 \end{matrix} \cdot \begin{matrix} p \\ \boxed{P_1} \\ p \end{matrix}$$

Step 2: Recursively Factor $B = L_1 U_1 P_1$ Let $D = C P_1^{-1}$

$$\begin{matrix} p \\ \boxed{U_1} \\ m/2 \\ \boxed{D} \\ m/2 \end{matrix} = \begin{matrix} m/2 & p-m/2 \\ \boxed{E} & \text{Rest of } U_1 \\ m/2 & \\ \boxed{F} & \text{Rest of } D \end{matrix}$$

Step 3: Partition First $m/2$ rows of U_1 and D

$$\begin{matrix} p \\ \boxed{A} \\ m \end{matrix} = \begin{matrix} m \\ \boxed{L_1} \\ m/2 \\ \boxed{FE^{-1}} \\ m/2 \\ \boxed{I} \end{matrix} \cdot \begin{matrix} p \\ \boxed{U_1} \\ m/2 \\ \boxed{G} \\ m/2 \end{matrix} \cdot \begin{matrix} p \\ \boxed{P_1} \\ p \end{matrix}$$

Step 4: Let $G = D - FE^{-1}U_1$ Let G' be the rightmost $p - m/2$ columns of G .

Step 5: Recursively Factor $G' = L_2 U_2 P_2$

$$\begin{array}{c} p \\ \square \\ p \end{array} P_3 = \begin{array}{cc} m/2 & p-m/2 \\ m/2 & \begin{array}{c} \square \\ \square \\ p-m/2 \end{array} \end{array}$$

Step 6: Construct P_3 from P_2 computed in Step 5.

$$\begin{array}{c} p \\ m/2 \ \square \ U_1 \\ m/2 \ \square \ G \end{array} = \begin{array}{c} m/2 \\ m/2 \end{array} \begin{array}{c} m/2 \\ m/2 \end{array} \begin{array}{c} \square \\ \square \\ m/2 \end{array} \begin{array}{c} p \\ \square \ H \\ \square \ U_2 \end{array} \cdot \begin{array}{cc} m/2 & p-m/2 \\ m/2 & \begin{array}{c} \square \\ \square \\ p-m/2 \end{array} \end{array}$$

Step 7: Let $H = U_1 P_3^{-1}$

$$\begin{array}{c} p \\ m \ \square \ A \end{array} = \underbrace{\begin{array}{cc} m/2 & m/2 \\ m/2 & \begin{array}{c} \square \\ \square \\ m/2 \end{array} \end{array}}_L \cdot \underbrace{\begin{array}{c} p \\ \square \ H \\ \square \ U_2 \end{array}}_U \cdot \underbrace{\begin{array}{c} p \\ \square \ P_3 \cdot P_1 \end{array}}_P$$

Step 8: Assemble Factors into $A = LUP$