CSC 721 Algorithms Fall 2017

Floyd-Warshall's Algorithm (All Points Shortest Path)[1]

Input: A weighted graph $G = (V, E)$ with weight function $f : V \times V \rightarrow \mathcal{R}^+ \cup \{+\infty\}$.
Let n denote the number of vertices in V. Number the vertices $V = \{v_1, v_2, v_3, ..., v_n\}$.

Output: An $n \times n$ matrix C such that $C_{i,j}$ is the cost of the shortest path from v_i to v_j.

Key Idea: Define a matrix $C_{i,j}^{(k)}$ as the cost of the shortest (restricted) path from v_i to v_j that goes through intermediate vertices numbered no higher than k.

Method:

for ($i = 1$; $i \leq n$; i++)
    $C_{i,i}^{(0)} = 0$
// end for i

for ($i = 1$; $i \leq n$; i++)
    for ($j = 1$; $j \leq n$; j++)
        if ($i \neq j$)
            $C_{i,j}^{(0)} = f(v_i, v_j)$
        // end if
    end for j
end for i

// End of initialization phase. Begin main loop.
for ($k = 1$; $k \leq n$; k++)
    for ($i = 1$; $i \leq n$; i++)
        for ($j = 1$; $j \leq n$; j++)
            $C_{i,j}^{(k)} = \min\left(C_{i,j}^{(k-1)} , (C_{i,k}^{(k-1)} + C_{k,j}^{(k-1)}) \right)$
        // end for j
    // end for i
// end for k
Let $C = C_{i,j}^{(n)}$
Return C.

[1] A version of this algorithm could be set to rock and roll music. That would be "Pink Floyd-Warshall".