Definitions for Asymptotic Measures

Let f and g be real-valued functions, over the natural numbers i.e, $f, g : \mathcal{N} \to \mathcal{R}$.

Big "O"

$f(n) \in \mathcal{O}(g(n))$ if and only if there exists $c > 0$ and there exists $N_0 > 0$ such that for all $n \geq N_0$, $|f(n)| \leq c|g(n)|$.

In some contexts we may drop the absolute value signs when we know both functions are non-negative.

Notice also we use the notation "$\in$" denoting set membership. In this context we are using the notation $\mathcal{O}(g(n))$ to denote the class of functions which are bounded above (in absolute value) by a multiple of $g(n)$.

We may also say "g dominates f" or "f is dominated by g", meaning $f(n) \in \mathcal{O}(g(n))$.

The use of the letter "O" comes from the phrase "order of", as in "Function f is order of g.

Little "o"

$f(n) \in \mathrm{o}(g(n))$ if and only if for all $c > 0$ there exists $N_0 > 0$ such that for all $n \geq N_0$, $|f(n)| \leq c|g(n)|$.

Note that this definition of "little o" differs from "Big O", only in the quantifier "for all c" instead of "there exists c".

Also note our definition of "little o" is equivalent to:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \;=\; 0$$

We may also say "g strictly dominates f" or "f is strictly dominated by g", meaning $f(n) \in \mathrm{o}(g(n))$.

Big Theta

$f(n) \in \Theta(g(n))$ if and only $f(n) \in \mathcal{O}(g(n))$ and $g(n) \in \mathcal{O}(f(n))$.

We may also say "f and g are co-dominant", meaning $f(n) \in \Theta(g(n))$.

Big Omega

$f(n) \in \Omega(g(n))$ if and only if there exists $c > 0$ and there exists $N_0 > 0$ such that for all $n \geq N_0$, $|f(n)| \geq c|g(n)|$.

In other words, $f(n) \in \Omega(g(n))$ if and only if $g(n) \in \mathcal{O}(f(n))$.

Wikipedia notes that this definition is due to D. E. Knuth (1976). The definition given by D. E. Knuth differs technically from an earlier definition given by G.H. Hardy and J.E. Littlewood (1914).

Big omega is used as an asymptotic lower bound, similar to the way Big "O" is used to denote an upper bound.