CSC702 Spring 2015
Theory of Computation – HW # 3

1. Given the following grammar G:

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow (\, E\,) \mid x \end{aligned}$$

   (a) Give a derivation for: $x + (\, x + x\,)$
   (b) Draw a parse tree for your derivation
   (c) Use the construction in the proof of Theorem 2.12 to construct (draw) a PDA which accepts $L(G)$.
   (d) Convert the grammar G to Chomsky Normal Form (CNF).

2. Prove (suggestion: use induction) that if G is a context free grammar in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

3. Show that the set of CFL are closed under the regular operations: `set union`, `concatentation`, and `Kleene closure`.

4. Show that the set of CFL are not closed under set intersection.

5. Is $L = \{a^n b^{2n} \mid n \geq 0\}$ a CFL ? Prove your result.

6. Let:

$$S = \{a^n \mid n \text{ is a perfect square }\}$$

   Use the pumping lemma for CFL to show that S is not a CFL.[1]

7. Extend the formal definition of a PDA to a 2-stack PDA. I.e., modify the definition of a 1-stack PDA:

$$P = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

   In your formal definition of a 2-stack PDA, define each member of the tuple (e.g., Q, Σ, etc.) used by your definition.

8. Show that a 2-stack PDA is "more powerful" than a 1-stack PDA. I.e., there exists a language which can be accepted by a 2-stack PDA and can not be accepted by a 1-stack PDA.

   Hint: Consider $\hat{L} = \{\, a^n b^n c^n \mid n \geq 0\, \}$.

   Note: For this problem, give an informal (English) description of how a two stack PDA can recognize $\hat{L}$.

[1] An integer n is a perfect square if and only if there exists an integer k such that $k^2 = n$.