1. Given the following grammar \( G \):

\[
E \rightarrow E + T \mid T \\
T \rightarrow (E) \mid x
\]

(a) Give a derivation for: \( x + (x + x) \)

(b) Draw a parse tree for your derivation

(c) Use the construction in the proof of Theorem 2.12 to construct (draw) a PDA which accepts \( L(G) \).

(d) Convert the grammar \( G \) to Chomsky Normal Form (CNF).

2. Prove (suggestion: use induction) that if \( G \) is a context free grammar in Chomsky normal form, then for any string \( w \in L(G) \) of length \( n \geq 1 \), exactly \( 2^n - 1 \) steps are required for any derivation of \( w \).

3. Show that the set of CFL are closed under the regular operations: set union, concatenation, and Kleene closure.

4. Show that the set of CFL are not closed under set intersection.

5. Is \( L = \{ a^n b^{2n} \mid n \geq 0 \} \) a CFL? Prove your result.

6. Let:

\[ S = \{ a^n \mid n \text{ is a perfect square} \} \]

Use the pumping lemma for CFL to show that \( S \) is not a CFL.\(^1\)

7. Extend the formal definition of a PDA to a 2-stack PDA. I.e., modify the definition of a 1-stack PDA:

\[ P = (Q, \Sigma, \Gamma, \delta, q_0, F) \]

In your formal definition of a 2-stack PDA, define each member of the tuple (e.g., \( Q, \Sigma, \) etc.) used by your definition.

8. Show that a 2-stack PDA is “more powerful” than a 1-stack PDA. I.e., there exists a language which can be accepted by a 2-stack PDA and can not be accepted by a 1-stack PDA.

*Hint: Consider \( \hat{L} = \{ \ a^n b^n c^n \mid n \geq 0 \}. \)

Note: For this problem, give an informal (English) description of how a two stack PDA can recognize \( \hat{L} \).

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\(^1\)An integer \( n \) is a perfect square if and only if there exists an integer \( k \) such that \( k^2 = n \).