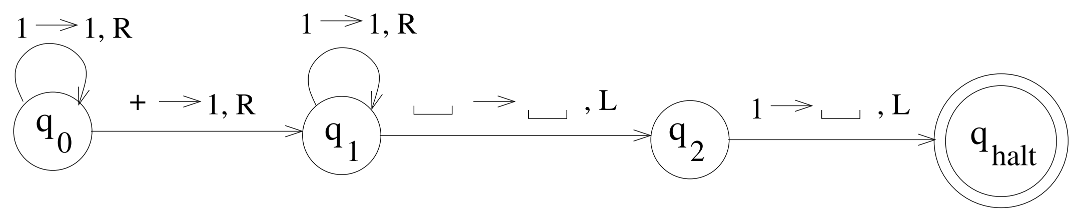
Example Turing Machines

Turing Machines to Define Computable Functions

Turing machines can be used to compute the value of a function. We assume that the input string is on the tape, and that the tape head position begins at the leftmost symbol of the input. The portion of the tape to the right of the input is blank. The computation finishes when the machine enters the state q_{halt}. The machine must leave the answer, and only the answer (i.e., the function value with extraneous symbols allowed) on the tape. The answer should begin at the the beginning of the tape.

In the example below, a Turing Machine is illustrated that computes the sum '$a + b$, where a and b are represented in "base 1". E.g., the number 3 is represented by "111". The number 7 is represented as "1111111". The input problem "3+5" is encoded as: "111+11111". We assume for this example that both a and b are positive integers, i.e., zeros are not allowed.

The alphabet for the machine illustrated below is $\Sigma = \{' ', '1'\}$.



Turing Machines as Language Deciders

In formal language theory, Turing Machines are used to develop a theory of computability. In this theory, a language is simply a set of strings over a finite alphabet. A Turing Machine decides a language L by processing an input string w and (after finitely many steps) entering exactly one of two possible halting states, named q_{accept} and q_{reject}. The state q_{accept} indicates that that the machine has decided the input $w \in L$. The state q_{reject} indicates that that the machine has decided the input $w \notin L$. An important result of this theory is that some languages are not decidable. I.e., no Turing Machine exists that decides if the input string is one of the strings in the language. Surprisingly, the theory also demonstrates languages which are not even recognizable; i.e., a language L so strange that a Turing machine does not exist which will accept all strings which are in L, even if the Turing machine might run forever on those strings which are not in L.

In the example below, the illustrated Turing Machine decides the language: $L = \{ a^n b^n \mid n \geq 0 \}$. All transitions which are not shown are assumed to go to state q_{reject}. $\Sigma = \{'a', 'b', ' '\}$.