CSC 222 **Algorithms** **Spring 2014**

Some $\mathcal{NP}$-complete problems

Definitions: Let $G = (V, E)$ be an undirected graph.

- A vertex cover of G is a subset $S \subseteq V$ such that each edge in G is incident on some vertex in S.
- A Hamiltonian circuit of G is a cycle in G that contains every vertex of V.
- A graph G is k-colorable if there exists an assignment of integers $\{1, 2, 3, ..., k\}$, i.e., the "colors", to the vertices of G such that no two adjacent vertices have the same color.
- The chromatic number of a graph G is smallest integer k such that G is k-colorable.
- Given a collection of sets $S_1, S_2, ..., S_n$, does there exist a sub-collection of k sets $S_{j_1}, S_{j_2}, S_{j_3}, ..., S_{j_k}$ such that
$$\bigcup_{i=1}^{n} S_i = \bigcup_{j=1}^{k} S_{k_j} \qquad ?$$

More Definitions:

Let $G = (V, E)$ be a directed graph.

- A feedback vertex set of G is a subset $S \subseteq V$ such that every cycle of G contains a vertex in S.
- A feedback edge set of G is a subset $F \subseteq E$ such that every cycle of G contains an edge in F.
- A directed Hamiltonian circuit of G is a cycle in G that contains every vertex of V.

Some $\mathcal{NP}$-complete problems

- Is a boolean expression satisfiable ?
- Does an undirected graph G have a clique of size k ?
- Does an undirected graph G have a vertex cover of size k ?
- Does an undirected graph G have a Hamiltonian circuit ?
- Is an undirected graph G k-colorable ?
- Does a directed graph G have a feedback vertex set of size k ?
- Does a directed graph G have a feedback edge set of size k ?
- Does a directed graph G have a directed Hamiltonian circuit ?
- Does a collection of sets $S_1, S_2, ..., S_n$ have a set cover of size k ?
- Knapsack: Given a set of integers $T = \{t_1, t_2, ...t_n\}$, and an integer K, does there exist a subset $S \subseteq T$ such that
$$\sum_{x \in S} x = K \qquad ?$$