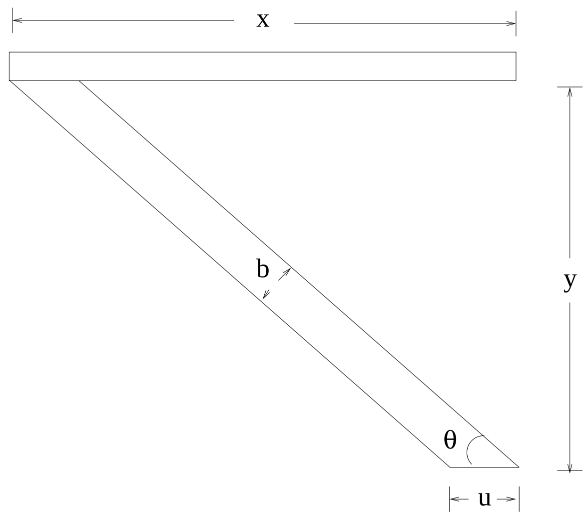
A Picnic Table Problem Solution
Solution by Henry Huang

Problem Statement:

Refer to the diagram below. The width of the table x and the height of the table (minus the thickness of the top) y are given. The width of the table leg b is also given. The thickness of the top is 1.5 inches. You are about to cut the leg (diagonal board) using a sliding mitre saw. But, you realize that you do not know the angle θ to set on the saw. Your task is to write a computer program that inputs three floating point numbers representing x, y and b, and prints the angle θ (in degrees) accurate to four significant digits.

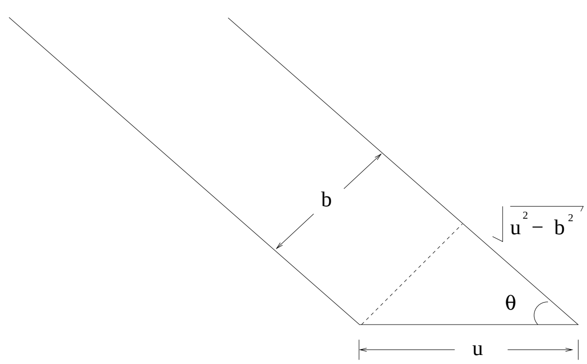


Solution:

Observe that if we knew u, we could consider the triangle formed by the floor, the lower edge of the table leg, and a vertical line drawn from the floor to the top end of the leg. In this triangle we see:

$$tan(\theta) = \frac{y}{x - u} \tag{1}$$

Also, consider a triangle at the base of the leg as illustrated below:



By Pythagorean theorem, we have three sides with lengths, b, u and $\sqrt{u^2 - b^2}$. By the definition of **tangent** we have:

$$tan(\theta) = \frac{b}{\sqrt{u^2 - b^2}} \tag{2}$$

Equating the right hand sides of equation (1) and (2), we have:

$$\frac{y}{x - u} = \frac{b}{\sqrt{u^2 - b^2}} \tag{3}$$

Notice equation (3) has only one unknown, u. Using simple algebra, we have:

$$y\sqrt{u^2 - b^2} = b(x - u) \quad . \tag{4}$$

Squaring both sides, we have:

$$y^2(u^2 - b^2) = b^2(x - u)^2 \tag{5}$$

or equivalently,

$$y^2u^2 - y^2b^2 = b^2x^2 - 2b^2xu + b^2u^2 \quad . \tag{6}$$

Collecting terms, and arranging them in order of decreasing exponent on u, we have the following quadratic equation in u:

$$(y^2 - b^2)u^2 + 2b^2xu - b^2(x^2 + y^2) = 0 \quad . \tag{7}$$

Using a standard formula for quadratic equations, and rejecting a negative solution we have:

$$u = \frac{-2b^2x + \sqrt{4b^4x^2 + 4(y^2 - b^2)b^2(x^2 + y^2)}}{2(y^2 - b^2)} \tag{8}$$

Finally, we find θ by:

$$\theta = sin^{-1}(b/u)$$