A Picnic Table Problem Solution
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Problem Statement:
Refer to the diagram below. The width of the table $x$ and the height of the table (minus the thickness of the top) $y$ are given. The width of the table leg $b$ is also given. The thickness of the top is 1.5 inches. You are about to cut the leg (diagonal board) using a sliding mitre saw. But, you realize that you do not know the angle $\theta$ to set on the saw. Your task is to write a computer program that inputs three floating point numbers representing $x$, $y$ and $b$, and prints the angle $\theta$ (in degrees) accurate to four significant digits.

Solution:
Observe that if we knew $u$, we could consider the triangle formed by the floor, the lower edge of the table leg, and a vertical line drawn from the floor to the top end of the leg. In this triangle we see:

$$\tan(\theta) = \frac{y}{x - u} \tag{1}$$

Also, consider a triangle at the base of the leg as illustrated below:
By Pythagorean theorem, we have three sides with lengths, \( b, u \) and \( \sqrt{u^2-b^2} \). By the definition of tangent we have:

\[
\tan(\theta) = \frac{b}{\sqrt{u^2-b^2}} \tag{2}
\]

Equating the right hand sides of equation (1) and (2), we have:

\[
\frac{y}{x-u} = \frac{b}{\sqrt{u^2-b^2}} \tag{3}
\]

Notice equation (3) has only one unknown, \( u \). Using simple algebra, we have:

\[
y\sqrt{u^2-b^2} = b(x-u) \ . \tag{4}
\]

Squaring both sides, we have:

\[
y^2(u^2-b^2) = b^2(x-u)^2 \tag{5}
\]

or equivalently,

\[
y^2u^2 - y^2b^2 = b^2x^2 - 2b^2xu + b^2u^2 \ . \tag{6}
\]

Collecting terms, and arranging them in order of decreasing exponent on \( u \), we have the following quadratic equation in \( u \):

\[
(y^2 - b^2)u^2 + 2b^2xu - b^2(x^2 + y^2) = 0 \ . \tag{7}
\]

Using a standard formula for quadratic equations, and rejecting a negative solution we have:

\[
u = \frac{-2b^2x + \sqrt{4b^4x^2 + 4(y^2 - b^2)b^2(x^2 + y^2)}}{2(y^2 - b^2)} \tag{8}
\]

Finally, we find \( \theta \) by:

\[
\theta = \sin^{-1}(b/u)
\]