CSC221 Spring 2012
Data Structures & Algorithms I
Exam # 2 Practice Problems
Scheduled date: Mon, April. 16, 2012

1. Describe how two data elements can be exchanged without using a temporary to store one of them. *Hint: Use the exclusive or operator.*

   Solution:

```
#include <iostream>
using namespace std ;

int main()
{
   int x, y ;

   x = 1 ; y = 2 ;
   x ^= y ;     // Exchange x and y using exclusive OR.
   y ^= x ;
   x ^= y ;
   cout << x << " " << y << endl ;  // Output is:  2 1
}
```

2. For this problem, refer to the attached copy of `token.h`, and `token.cc`. In our project on syntax trees, we only allowed variables to be a single letter. Modify class `token` and function `get_next()` to allow multiple-character variable names up to 31 characters in length. *Hint: use a fixed-length array of type* `char` *; be sure to remember to null-terminate C-style character strings.*

   Solution: follows:

```
#define MAX_VAR 32

union extra_info {
   char variable[MAX_VAR] ;  //  Make 'variable' a null-terminated
   int number   ;            //  C-style string.
} ;
```

   We also change the in-line function `get_variable()` to:

```
char * get_variable() { return &(details.variable[0]) ; }
```

   In the file "token.cc", we change the following section of `get_next()`

   Old code:

```
if ( isalpha(ch) ) {
      what_type = vari ;
      details.variable = ch ;
}
```

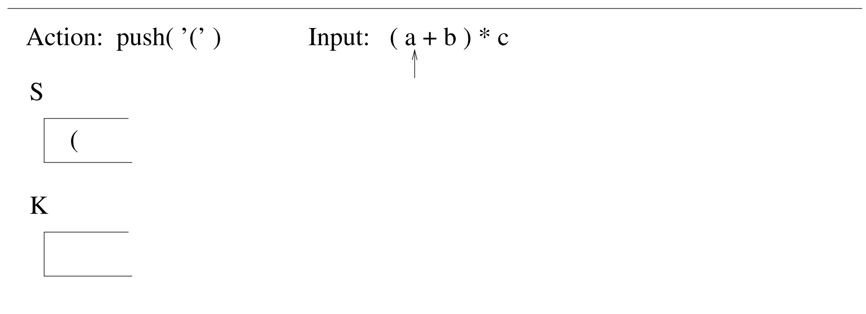
New code:

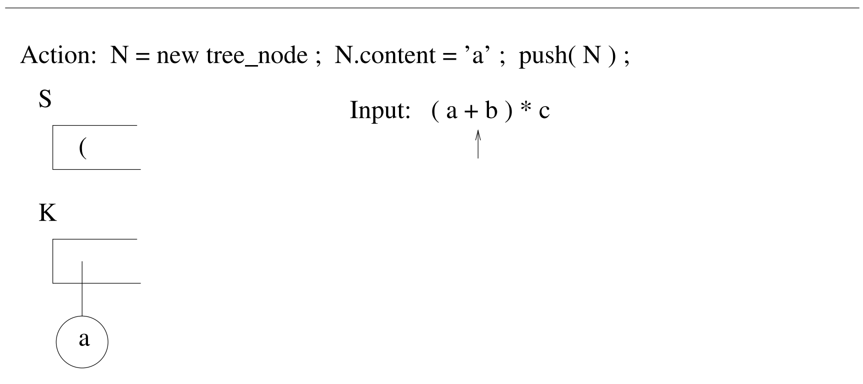
```
if ( isalpha(ch) ) {
      what_type = vari ;
      details.variable[0] = ch ;
      int ch_count = 1 ;
      do {
         ch = buffer[pos] ; pos++ ;
         if ( isalpha(ch) ) {
            details.variable[ch_count] = ch ;
            ch_count++ ;
         }
      } while ( isalpha(ch) ) ;
      pos-- ; // Put this character back.
      details.variable[ch_count] = '\0' ;  // Null terminate the string.
}
```

3. Describe the steps taken by an operator precedence parser to process the expression `(a + b) * c` and build a syntax tree. Draw a sequence of diagrams illustrating the contents of the operator stack, the tree stack, the current character, and the actions taken by the parser.

Solution: See the sequence of diagrams below for the actions taken by an operator precedence parser on input `(a + b) * c`.

Input: (a + b) * c   Initially, input character is '('

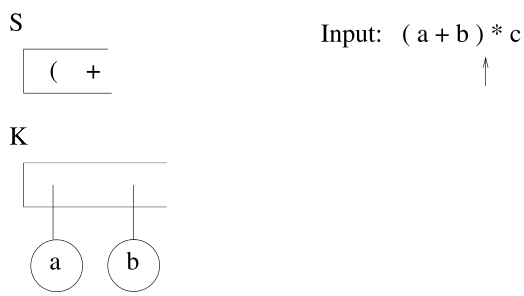




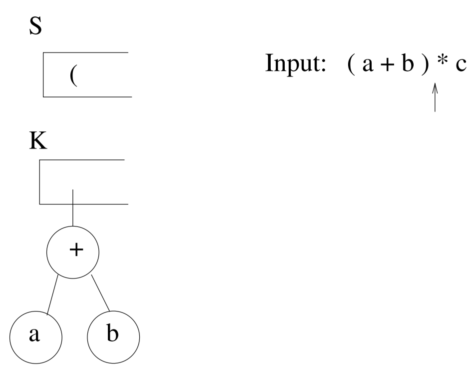
Action: push(+)

Input: (a + b) * c

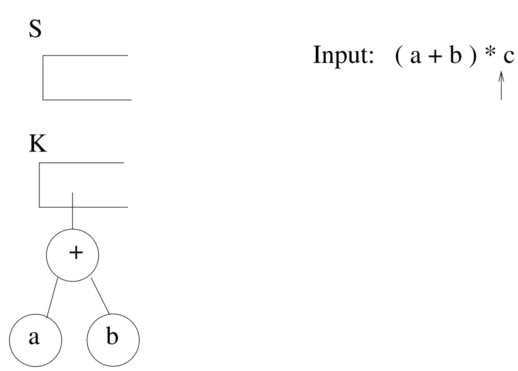
Action: N = new tree_node ; N.set_content(’b’) ; push(N) ;



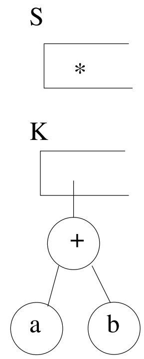
Action: Pop all operations to matching ’(’ and build tree.



Action: Pop the ’(’ and discard the matching ’)’.

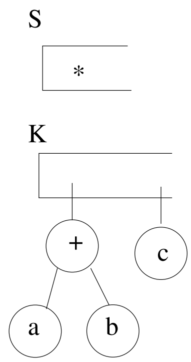


Action: push(*)



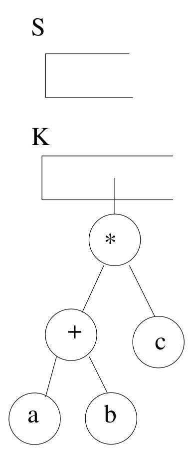
Input: (a + b) * c

Action: N = new treenode ; N.set_content('c') ; push (N) ;



Input: (a + b) * c

Action: Pop remaining ops on S and build tree



Input: (a + b) * c

Parse complete.

4. Consider the following class `two_d_point`

```
class two_d_point {
   private:
      double x ;
      double y ;
   public:
      two_d_point() { x = 0.0 ; y = 0.0 ; }
      two_d_point(double a, double b) { x = a ; y = b ; }
      ~two_d_point() { }
      // Add operator "<" here.
} ;
```

Add the operator "less than" (symbol $<$) to the class `two_d_point`. Add the appropriate declaration to the class declaration (above). Give an implementation, written outside the class using the scope resolution operator. **Note:** A point a is "less than" point b if it is closer to the origin $(0, 0)$.

Solution:

```
class two_d_point {
   private:
      double x ;
      double y ;
   public:
      two_d_point() { x = 0.0 ; y = 0.0 ; }
      two_d_point(double a, double b) { x = a ; y = b ; }
      ~two_d_point() { }
      // Add operator "<" here.
      bool operator<( two_d_point b ) ;
} ;

bool two_d_point::operator<( two_d_point b )
{
   double d0, d1 ;

   d0 = sqrt( x*x + y*y ) ;  // Distance for left operand.
   d1 = sqrt( b.x*b.x + b.y*b.y ) ;  // Distance for right operand.
   return (d0 < d1) ;
}
```

5. Given the following declaration for a circular-array implementation of a queue of integers:

```
#define MAXQ 100
class queue {
  private:
    int head ;
    int tail ;
    int q[MAXQ] ;
  public:
    queue() { head = 0 ; tail = 0 ; }
    ~queue() { }
    void enqueue( int x ) ;
    int dequeue( ) ;
    bool is_full( ) ;
    bool is_empty( ) ;
} ;
```

(a) Write implementations for the functions `enqueue()`, `dequeue()`, `is_full()`, and `is_empty()`.

Solution:

```
#include <iostream>
#include <cstdlib>
using namespace std ;

  #define MAXQ 100
   class queue {
     private:
       int head ;
       int tail ;
       int q[MAXQ] ;
     public:
       queue() { head = 0 ; tail = 0 ; }
       ~queue() { }
       void enqueue( int x ) ;
       int dequeue( ) ;
       bool is_full( ) ;
       bool is_empty( ) ;
   } ;

  void queue::enqueue( int x )
  {
     if ( is_full() ) {
        cerr << "enqueue():  Error: queue is full." << endl ; exit(1) ;
     }
     else {
       q[tail] = x ;   tail = (tail + 1) % MAXQ  ;
     }
  }
```

```
int queue::dequeue( )
{
   int r ;
   if ( is_empty() ) {
      cerr << "dequeue():  Error: queue is empty." << endl ; exit(2) ;
   }
   else {
     r = q[head] ;   head = (head + 1) % MAXQ  ;
     return(r) ;
   }
}

// When tail is advanced, and wraps around to hit the head,
// then the queue is considered full.
bool queue::is_full( )
{
   return ((( tail + 1 ) % MAXQ ) == head )  ;
}

// When head and tail are equal, we consider the queue to be empty.
bool queue::is_empty( )
{
   return ( tail == head )  ;
}
```

6. Illustrate (using a sequence of diagrams) the results of inserting the following numbers into a 2-3 tree (of integers):

   `22 12 30 18 17 8 24 32 19`

   Solution: See a solution as a separate document, **http://menehune.opt.wfu.edu/CSC221**

7. A 2-3 tree with two levels can have a maximum of 8 data items. Consider an arbitrary sequence of eight input numbers used to build a 2-3 tree. Which of the following is true ?

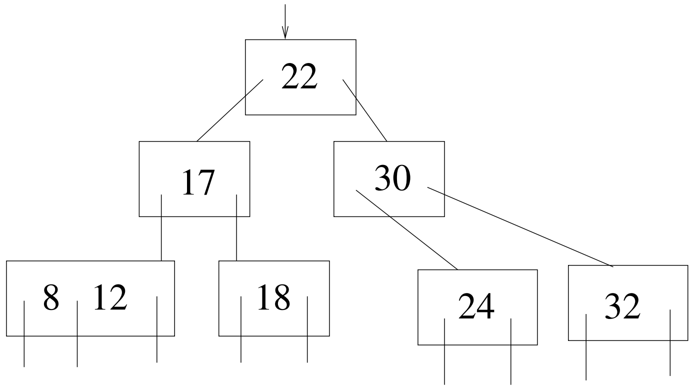
   (a) Eight numbers will always result in a 2-3 tree with two levels.

   (b) The number of levels will depend on the order in which the numbers are inserted.

   Solution: Alternative 7a is false; alternative 7b is true. The number of levels depends on the numbers involved and the order in which they are inserted. Consider the first eight numbers in problem 6 :

   `22 12 30 18 17 8 24 32`

   A 2-3 tree built with these numbers (in this order) is given below:



8. Illustrate (using a sequence of diagrams) the result of inserting the following words in a trie:

```
carrot car cat catch chill check apple dog dock deck
```

Solution: See the solution given in class.

9. Intro to complexity analysis:

   (a) Give the definition of Big $\mathcal{O}$ notation. I.e., what do we mean (precisely) when we write:

   $$f(n) \in \mathcal{O}(g(n))$$

   Solution:
   $f(n) \in \mathcal{O}(g(n))$ means there exists $c > 0$ and an $N_0 > 0$ such that $|f(n)| \leq c|g(n)|$ for all $n \geq N_0$.

   (b) Give the definition of little o notation: I.e., what do we mean (precisely) when we write:

   $$f(n) \in \mathrm{o}(g(n))$$

   Solution: $f(n) \in \mathrm{o}(g(n))$ means for all $c > 0$, there exists an $N_0 > 0$ such that $|f(n)| \leq c|g(n)|$ for all $n \geq N_0$. An equivalent and more compact definition is:

   $$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

   (c) Derive an asymptotic upper bound (Big $\mathcal{O}$) for the following iterative code segment. Assume that n is a power of two; i.e., $n = 2^k$ for some $k \geq 0$.

```
t = 0 ;
while ( n > 1 ) {
   m = n ;
   while ( m > 1 ) {
      t ++ ;
      m = m / 2 ; // Integer divide.
   }
   n = n / 2 ; // Integer divide.
}
```

   Solution: Given in class.

(d) Derive an asymptotic upper bound (Big $\mathcal{O}$) for the following recursive code segment. Assume that n is a power of two; i.e., $n = 2^k$ for some $k \geq 0$.

```
int f( int n )
{
   if ( n == 1 ) return 1 ;
   else {
      int t = 0 ;
      for ( int i = 0 ; i < n ; i++ ) t += i ;
      return t + f( n/2 ) ;
  }
}
```

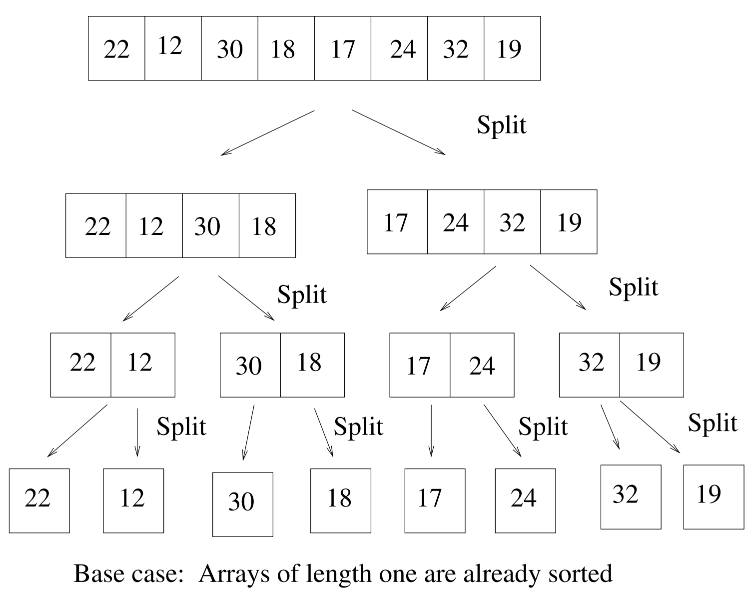
Solution: Given in class.

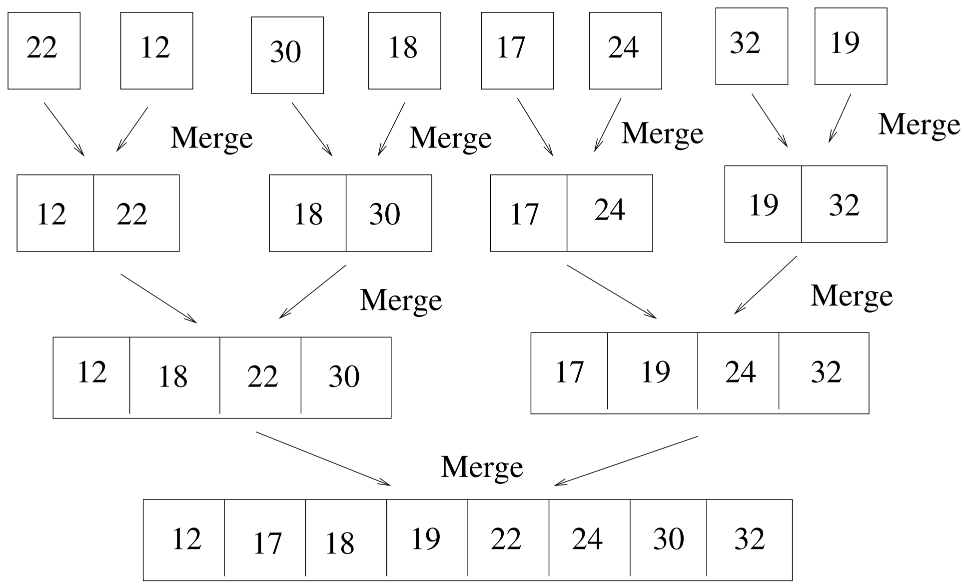
10. Mergesort

Draw a tree illustrating the sequence of function calls (and the returned results) when mergesort is applied to:

22	12	30	18	17	24	32	19

Solution: On the way down the recursive levels, `mergesort()` splits the array. In the base case, each array of length one are already sorted. On the way back up from the recursive levels, the intermediate results are merged.

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